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Demand System Estimation

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Plan of the talk

1. Single product demand
 - Homogenous goods
 - IV estimation (2SLS)
1. Differentiated Products Demand System
 - Almost Ideal Demand System (AIDS)
1. An application: Campina/Friesland

Demand System Estimation

- This talk will focus on the estimation of market demand function
- The relevance of demand estimation in antitrust analysis is evident: market definition, merger simulation, estimation of conduct parameters
- Quite vast literature and techniques: continuous choice models, discrete choice models
- In this talk we will deal with the most popular continuous choice model for differentiated products, the AIDS model (Deaton and Muellbauer, 1980)
- But, before that, we start with a primer on homogenous goods

Demand System Estimation: single product demand

- Simplest case : one equation to estimate
- Relevant to understand how “econometrics” can never be used in isolation
- Product is homogenous: consumers do not care about the brand
- Think of sugar, oil, corn, steel

Single product demand

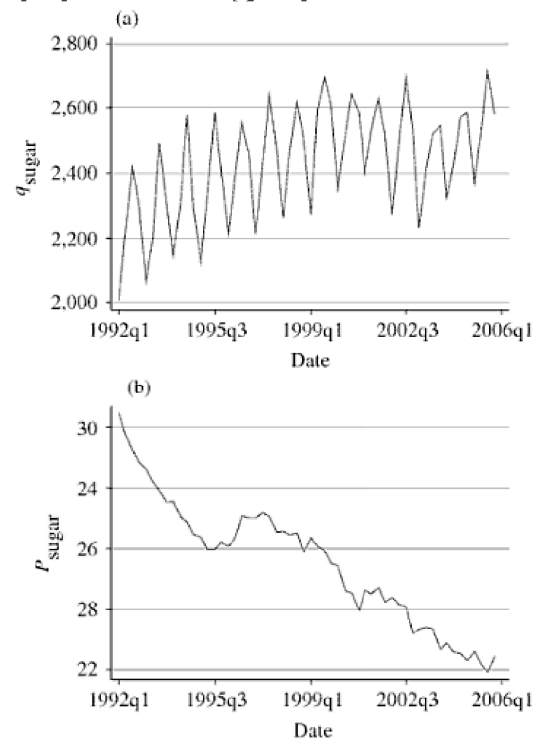
- Consider the log-linear demand function:

$$Q_t = D(P_t) = e^{a+\xi_t} P_t^{-b},$$

- ξ_t is the error component, unknown to the econometrician; assumptions on this crucial
- For example, if we assume it is known by the firms, this generates correlation with P (prices): endogeneity bias
- Take logs:
$$\ln Q_t = a - b \ln P_t + \xi_t.$$
- b is the market demand elasticity: our parameter of main interest
- Data requirements: quantities, prices, instruments if necessary

Single product demand

- We start by plotting quantities and data:



- Quantities: seasonality, average increase, cyclical downturn 2002
- Prices: downward trend

Single product demand

- Clear negative relationship between prices and quantities:
 - But does that imply a causal link between reduced prices and increased demand?
- Our simple model is clearly misspecified.
 - Only source of quantity variation is prices: we know that substantial quarterly variation in Q not explained by P .
- Not only this: also the average trend (consistent with price movement) can be explained by other factors:
 - Consumers became more health conscious, or richer...
- This means we need to incorporate other features in the model (determined outside the econometrics realm)

Single product demand

- Of course, supply factors may also affect deliveries.
 - To be sure we identify demand, we must be confident that OLS assumptions are satisfied

$$E[\xi_t(\theta^*) \mid P_t] = 0$$

- A way to proceed is to plot residuals against the regressors; plot residuals over time (this would show seasonality)
 - We can then introduce seasonal (3) dummies to account for that. Why 3? Otherwise we have perfect collinearity with the constant term
- Regression of model with seasonal dummies on US data (1992–2006) returns an elasticity of -0.38 . Fairly inelastic demand: should we draw policy conclusions?

Single product demand

- The OLS assumption can fail for numerous reasons
 - Non price drivers of the unobserved component of demand may also impact on prices.
 - In an extreme case in which supply is stable, such demand variation will trace out the supply curve instead of demand.
 - More in general, omitted variable bias
- Endogeneity problem. Solution: Instrumental Variables techniques. Two basic requirements of the instruments:
 - Correlated with the potentially endogenous regressor
 - uncorrelated with the unobserved component of demand
- Popular estimator: 2SLS, gmm

Single product demand

- 2SLS estimator can be obtained in one step. However, useful to look at two stage estimation procedure
- First stage: regression of endogenous variable(s) on instrument and exogenous variables
- First stage helps assess the first condition required for an instrument
- Second condition harder to test: tests exist based on the observed correlation between the residuals and the endogenous variable
- Our example: instrument price of sugar with farmer wages. Cost component, they affect prices but unlikely to affect demand (farmers residual part of population)

Single product demand

- The 2SLS estimation proceeds in two stages

1st-stage regression: $\ln P_t = a - b \ln W_t + \gamma_1 q_{1t} + \gamma_2 q_{2t} + \gamma_3 q_{3t} + \varepsilon_t$,

2nd-stage regression: $\ln Q_t = a - b \ln \widehat{P}_t + \gamma_1 q_{1t} + \gamma_2 q_{2t} + \gamma_3 q_{3t} + v_t$,

- Quarterly dummies included in the first stage
- Estimates of elasticity using IV: -0.27 , below previous estimates
- Results in IV estimation need to be carefully scrutinised: two conditions need to be fulfilled.
 - Conditional expectation of the error equal 0
 - Significant correlation between instrument and endogenous
- First condition extremely difficult to test: can we residuals, but such residuals constructed with the assumption
- A variety of potential IV results can be tested against each other when the model is overidentified, but the bottom-line is that the first assumption must be theoretically founded.

Single product demand

- Second condition easier to test from the first stage regression
- We want the instrument (wages) to have a strong explanatory power of the endogenous (prices)
- In our example: correlation significant, but negative!!!!
- If we look at data, wages go up (while prices go down). This means that, even if wages are a driving force of prices, they're certainly not the major one.
- We should then ask: why prices are going down? Raw materials, institutional factors, demand factors?
- This helps us in: better specifying the model, find more suited instruments
- That's why econometrics cannot work in isolation

Demand System Estimation: differentiated products demand system

- We now turn to the analysis of differentiated product markets
- Think of products where brand is relevant (shampoos, milk, yoghurt, pasta)
- Consumers have different relative preferences for goods with different prices
- Differentiated product demand systems are then estimated as systems of individual product demand equations
- Demand for a product depends on its own price and on the price of substitutes

Demand System Estimation: differentiated products demand system

- Consider the popular log-linear differentiated product demand system

$$\ln Q_{1t} = a_1 - b_{11} \ln P_{1t} + b_{12} \ln P_{2t} + \dots + b_{1J} \ln P_{Jt} + \gamma_1 \ln y_t + \xi_{1t},$$

$$\ln Q_{2t} = a_2 - b_{21} \ln P_{1t} + b_{22} \ln P_{2t} + \dots + b_{2J} \ln P_{Jt} + \gamma_2 \ln y_t + \xi_{2t},$$

$$\vdots$$

$$\ln Q_{Jt} = a_J - b_{J1} \ln P_{1t} + b_{J2} \ln P_{2t} + \dots + b_{JJ} \ln P_{Jt} + \gamma_J \ln y_t + \xi_{Jt}.$$

- This system may be derived by utility maximization subject to a budget constraint: demand depends on income and prices
- With aggregate data, we might use GDP
- Since we look at specific industries, the problem is usually reformulated as a two-stage decision:
 - first decision, how much budget to spend on a category
 - second decision, how to allocate the budget

Demand System Estimation: differentiated products demand system

- Using two-stage interpretation, expenditure can be used instead of income.
- Demand equations will then be termed conditional demand equations
- Example: Hausman (1994) estimates a three-level choice model. Consumers choose:
 - the level of expenditure on beer
 - how to allocate the expenditure among three categories
 - how to allocate the expenditure among the various brands of beer
- At each level, you need to construct an appropriate price index (could be weighted by expenditure share)

Demand System Estimation: differentiated products demand system

- Notice: elasticities (for example second stage) are conditional on given level of expenditure
- Need to take this into account for example for market definition (there is no income effect)
- For this Hausman estimates a first level equation, where aggregate demand is regressed on aggregate price and demographics
- Finding instruments in the context of differentiated products demand is difficult
 - We need a lot of them
 - Cost related variables, they are not product specific (generally)
- Potential solution proposed by Hausman: he estimates demand using city-level prices, and uses prices in other cities as instruments

Demand System Estimation: differentiated products demand system

- The logic: if (very big if)...
- demand shock are city specific and independent across cities
- cost shocks are correlated across markets
- ... Then any correlation between the price in this market and the prices in other markets will be due to cost movements
- And we have valid instruments
- Another potentially satisfactory instrument would be the price of a good that shares the costs but which is not a substitute or complement
- Think of goods for which oil is a major cost component (jet fuel and oil for heating)

Demand System Estimation: differentiated products demand system

- Log linear demand easy to estimate
- However, they impose considerable restrictions on the structure of consumer preferences
- Constant own and cross price elasticities of demand
- Plus, potentially serious internal consistency issue when we use aggregate data
- If we only include aggregate income variable, estimates may suffer from aggregation bias (show example)

Demand System Estimation: differentiated products demand system

- Study of aggregability conditions has provided motivation for many of the most popular demand systems model, as the AIDS
- Before looking at that model, let's recall some useful contribution from choice theory
- Recall indirect utility function

$$V(p, y; \theta) = \max_q u(q; \theta) \quad \text{subject to } pq \leq y$$

- so that $V(\cdot)$ represents the max utility that can be achieved for a given set of prices and income
- Roy's identity:

$$q_j(p, y; \theta) = - \frac{\partial V(p, y; \theta)}{\partial p_j} \bigg/ \frac{\partial V(p, y; \theta)}{\partial y}$$

Demand System Estimation: differentiated products demand system

- Practical implication: we just have to write an IUF and differentiate it to get a parametric demand systems
- We use a version of Roy's identity that uses expenditure shares
- Restate Roy's identity

$$w_j(p, y; \theta) \equiv \frac{p_j q_j(p, y; \theta)}{y} = \left(-p_j \frac{\partial V(p, y; \theta)}{\partial p_j} \right) / \left(y \frac{\partial V(p, y; \theta)}{\partial y} \right) = \left(-\frac{\partial V(p, y; \theta)}{\partial \ln p_j} \right) / \left(\frac{\partial V(p, y; \theta)}{\partial \ln y} \right)$$

- Estimating a model using the expenditure share on a good provides the same info that using quantity demanded
- Having logs provides algebraically more convenient model

AIDS (Almost Ideal Demand System)

- One of the most used model
 - It satisfies nice aggregability conditions
 - If we take a lot of consumers behaving as predicted by AIDS and aggregate their demand, we still have an AIDS demand system
 - Relevant parameters easy to estimate
 - Data needed, prices and expenditure shares, usually available for the analyst

- In AIDS, the indirect utility function is assumed to be:

$$V(p, y; \theta) = \frac{\ln y - \ln a(p)}{\ln b(p) - \ln a(p)}$$

- $a(p)$, $b(p)$, parametric functions of underlying price data

$$\ln a(p) = \alpha_0 + \sum_{k=1}^J \alpha_k \ln p_k + \sum_{k=1}^J \sum_{j=1}^J \gamma_{jk} \ln p_k \ln p_j$$

$$\ln b(p) = \ln a(p) + \beta_0 \prod_{k=1}^J p_k^{\beta_k}$$

Demand System Estimation: AIDS

- Applying Roy's identity for the expenditure share for product j gives

$$w_j(p, y; \theta) = \left(-\frac{\partial V(p, y; \theta)}{\partial \ln p_j} \right) / \left(\frac{\partial V(p, y; \theta)}{\partial \ln y} \right) = \alpha_j + \sum_{k=1}^J \gamma_{jk} \ln p_k + \beta_j \ln \left(\frac{y}{P} \right)$$

- Where P can be thought of as a price that deflates income

$$\ln P = \alpha_0 + \sum_{k=1}^J \alpha_k \ln p_k + \sum_{k=1}^J \sum_{j=1}^J \gamma_{jk} \ln p_k \ln p_j$$

- In practice, this price is replaced by a Stone price index, that makes expenditure shares linear in the parameters.

$$\ln P = \sum_{k=1}^J w_j \ln p_j.$$

Demand System Estimation: AIDS

- In practice, an AIDS system can be implemented in this way
 - calculate w_{jt} using the price of j at time t , the quantity demanded of j at time t and total expenditure.
 - Calculate the Stone price index
 - Run the following linear regression:

$$w_{jt} = \alpha_j + \sum_{k=1}^J \gamma_{jk} \ln p_{kt} + \beta_j \ln \left(\frac{y_t}{P_t} \right) + \xi_{jt},$$

- retrieve the $J+2$ parameters of interest
- The own and cross price elasticities can be retrieved from the AIDS parameters by noting that

$$\ln w_j = \ln p_j + \ln q_j - \ln y \quad \Longleftrightarrow \quad \ln q_j = \ln w_j - \ln p_j + \ln y,$$

Demand System Estimation: AIDS

- so that demand elasticities:

$$\eta_{jk} = \begin{cases} \frac{\partial \ln q_j}{\partial \ln p_k} = \frac{\partial \ln w_j}{\partial \ln p_k} - 1 & \text{if } j = k \\ \frac{\partial \ln q_j}{\partial \ln p_k} = \frac{\partial \ln w_j}{\partial \ln p_k} & \text{if } j \neq k \end{cases}$$

- Differentiating the AIDS expenditure share equation:

$$\frac{\partial \ln w_j}{\partial \ln p_k} = \frac{\gamma_{jk} - w_k \beta_j}{w_j}$$

- so, own and cross elasticities depend on both the parameters and expenditure shares

$$\eta_{jk} = \begin{cases} \frac{\gamma_{jk} - w_k \beta_j}{w_j} - 1 = \frac{\gamma_{jk}}{w_k} - \beta_j - 1 & \text{if } j = k \\ \frac{\gamma_{jk} - w_k \beta_j}{w_j} = \frac{\gamma_{jk}}{w_j} - \frac{w_k}{w_j} \beta_j & \text{if } j \neq k \end{cases}$$

Demand System Estimation: AIDS

- Slightly dangerous character of these formulas
- suppose little variation in the data, coefficients estimated to be close to 0, own elasticity close to -1 and cross elasticities close to 0
- results are not implausible, but they are simply drawn by the imprecision of the estimates
- The system can be estimated equation by equation, but most stat. Packages provide easy command for simultaneous estimation (stata reg3)
- To render estimation more tractable, parameter restrictions can be imposed from the theory

Demand System Estimation: AIDS

- Suppose we have two goods. We have two simultaneous demand equations

$$Q_1 = a_1 - b_{11}p_1 + b_{12}p_2 + c_1y \quad \text{and} \quad Q_2 = a_2 - b_{21}p_1 + b_{22}p_2 + c_2y.$$

- First restriction can be derived from choice theory: Slutsky symmetry (total substitution effect) is symmetric across any pair of goods

$$\frac{\partial Q_1}{\partial p_2} + Q_2 \frac{\partial Q_1}{\partial y} = \frac{\partial Q_2}{\partial p_1} + Q_1 \frac{\partial Q_2}{\partial y}.$$

- This property is derived from the rational individual utility maximization conditions
- In the simple two products model it implies that
 - $b_{21} = b_{12}$ and $c_1 = c_2 = 0$

Demand System Estimation: AIDS

- In general a set of sufficient conditions for Slutsky symmetry to be fulfilled are:

$$\frac{\partial Q_1}{\partial y} = \frac{\partial Q_2}{\partial y} = 0 \quad \text{and} \quad \frac{\partial Q_1}{\partial p_2} = \frac{\partial Q_2}{\partial p_1}.$$

- We are saying two things
 - Income effects are negligible
 - symmetry in the cross-price demand derivative
- implication of the assumptions: we have fewer parameters to estimate
- sadly, aggregate demand systems will not in general satisfy Slutsky symmetry
- For intuitive reasons:

$$\frac{\partial Q_{\text{Virgin}}}{\partial p_{\text{Coke}}} \neq \frac{\partial Q_{\text{Coke}}}{\partial p_{\text{Virgin}}}$$

Demand System Estimation: AIDS

- The aggregate cross price elasticities of two products will not generally be symmetric even if Slutsky symmetry is satisfied

$$\eta_{12} = \frac{\partial \ln Q_1}{\partial \ln P_2} = \frac{\partial Q_1}{\partial P_2} \frac{P_2}{Q_1} = \frac{P_2}{Q_1} b_{12},$$

$$\eta_{21} = \frac{\partial \ln Q_2}{\partial \ln P_1} = \frac{\partial Q_2}{\partial P_1} \frac{P_1}{Q_2} = \frac{P_1}{Q_2} b_{21},$$

- Choice theory suggests that individual demand functions will be homogenous of degree 0:

$$q_i(\lambda p_1, \dots, \lambda p_J, \lambda y) = q_i(p_1, \dots, p_J, y).$$

- This derives from the b.c in a utility maximization problem. Indeed, note that the two problems...

$$\max_q u(q) \text{ subject to } \sum_j p_j q_j = y \quad \text{and} \quad \max_q u(q) \text{ subject to } \sum_j \lambda p_j q_j = \lambda y.$$

- ...are fully equivalent

Demand System Estimation: AIDS

- This assumption survives aggregation provided it is interpreted in the right way
- homogeneity of degree 0 implies that:

$$w_j(\lambda p_1, \dots, \lambda p_J, \lambda y) = w_j(p_1, \dots, p_J, y) \quad \text{for } \lambda > 0.$$

- recall that in the AIDS model:

$$w_{jt}(p, y) = \alpha_j + \sum_{k=1}^J \gamma_{jk} \ln p_{kt} + \beta_j \ln \left(\frac{y_t}{P_t} \right) + \xi_{jt},$$

- So that the homogeneity restriction implies:

$$w_{jt}(\lambda p, \lambda y) = \alpha_j + \sum_{k=1}^J \gamma_{jk} \ln \lambda p_{kt} + \beta_j \ln \left(\frac{\lambda y_t}{P_t(\lambda)} \right) + \xi_{jt} = w_{jt}(p, y)$$

Demand System Estimation: AIDS

- This in turn implies that the following parameters' restrictions must hold

$$\sum_{j=1}^J \alpha_j = 1, \quad \sum_{j=1}^J \gamma_{jk} = 0, \quad \sum_{k=1}^J \gamma_{jk} = 0,$$

- where do these restrictions come from?

$$\begin{aligned} \sum_{k=1}^J \gamma_{jk} \ln \lambda p_{kt} &= \sum_{k=1}^J \gamma_{jk} (\ln \lambda + \ln p_{kt}) \\ &= (\ln \lambda) \left(\sum_{k=1}^J \gamma_{jk} \right) + \sum_{k=1}^J \gamma_{jk} \ln p_{kt} = \sum_{k=1}^J \gamma_{jk} \ln p_{kt}, \end{aligned}$$

- where the latter equality holds if $\sum_{k=1}^J \gamma_{jk} = 0$

Demand System Estimation: AIDS

- Another restriction is the additivity restriction: the demands must satisfy the budget constraint

$$\sum_{j=1}^J p_j q_j = y, \quad \text{where } q_j = q_j(p, y),$$

- This provides cross equation restrictions: In AIDS model

$$\begin{aligned} \sum_{j=1}^J w_{jt}(p, y) &= \sum_{j=1}^J \left(\alpha_j + \sum_{k=1}^J \gamma_{jk} \ln p_{kt} + \beta_j \ln \frac{y_t}{p_t} + \xi_{jt} \right) \\ &= \sum_{j=1}^J \alpha_j + \sum_{j=1}^J \ln p_{kt} \left(\sum_{j=1}^J \gamma_{jk} \right) + \ln \left(\frac{y_t}{p_t} \right) \sum_{j=1}^J \beta_j + \sum_{j=1}^J \xi_{jt} = 1 \end{aligned}$$

- necessary conditions for our expenditure share system to always this condition gives us the additivity restrictions

$$\sum_{j=1}^J \alpha_j = 1; \quad \sum_{j=1}^J \gamma_{jk} = 0; \quad \sum_{j=1}^J \beta_j = 0$$

The Campina–Friesland case

- We now turn to a real–case application of the model
- We look at a merger case, the one between Campina and Friesland, two dutch producers of dairy products
- The companies were active both in the procurement of raw milk, and in the sale of final products
- A long list of products involved (milk, yoghurt, vla, cream...)
- Market is national in scope for most products, wider for other (long life drinks, or pharmaceutical products)
- The merger was cleared with remedies, both structural and behavioural (concern were both in the wholesale and the retail market)

Campina/Friesland

- The econometrics exercise, which in the end proved to be quite controversial (the Commission basically decided to disregard it) had a twofold purpose:
 - To understand the boundaries of the markets (the product dimension)
 - The competitive constraints between the merging firms and other competitors (included the private labels)
- We are going to see now some details of the application.
- We don't go again on the model, although there might be some non relevant variation between what we have seen so far and the real application

Campina/Friesland

- So, data (source IRI) have been collected for different products at different level of aggregations (regional, supermarket, and Infoscan)
- $8 \times 3 = 24$ databases have been created
- Products: milk, yoghurt, vla and buttermilk, chocolate milk, flavoured milk (fresh and long life), drinkyoghurt
- Data were collected on product characteristics, volume and value of sales
- Info on products sold under promotions (several dummy variables were created), different packaging
- Several aggregations were made to make the estimation tractable

Campina/Friesland

- Data were already aggregated at weekly level at a retail level
- Aggregation had to be done between different flavours and package size.
- This is crucial: you make a simplification needed not to have too many parameters (you wouldn't need to do it using discrete choice models)
- Also data in private labels have been aggregated
- Aggregating data across chains is dangerous since the average price is not faced by any consumer
- However, market demand elasticity can be overestimated when you use chain level data . You might estimate a high elasticity of demand, while what happens is just a redistribution of sales across chains (so market demand elasticity is 0)

Campina/Friesland

- The Commission concludes that if the analyst is trying to infer the market level demand for a product and consumers are likely to switch across retailers when prices change, then she should use market level data, otherwise if such switches are unlikely, then chain level data are preferable
- Now, let us turn to more specific issues in the modeling strategy. As said, we don't go through the details of the AIDS specification, but few issues are worth mentioning
- First issue: wrong signs.
- In an AIDS system, estimated cross price elasticities are not guaranteed to be positive.
- How to act: first, are product complements?

Campina/Friesland

- Second: are coefficients statistically different from 0? If not, you can constrain the coefficients to be zero in the estimation if you need elasticities to do merger simulation or any kind of welfare analysis
- If elasticities are negative and statistically significant, the appropriate response depends on the number of negative elasticities over the total number of cross price elasticities estimated.
- With a large number of estimated elasticities, it is not surprising that some of them are negative
- However, if they are too many, one should reconsider the estimated model
- The second issue: endogeneity

Campina/Friesland

- The use of price data from one region to instrument for the price of another region.
- This approach does not appear particularly adequate in the context of the present case since the necessary assumption that demand shocks across regions or retailer chains are independent are not likely to be met
- Consideration: prices are not endogenous in the present case. They are indeed set by retailers prior to consumers making their purchase decisions. Retailers set the price and at that price their supply is perfectly elastic.
- In other words, regardless of quantity demanded prices are ultimately determined by costs factors during the measurement period.

Campina/Friesland

- Assumption which is hard to justify. Question: do retailers foresee demand shocks (think of advertising campaigns)?
- Other major problems: omitted variable bias. They control for time invariant heterogeneity using fixed effects estimates.
- The empirical implementation
- The IRI dataset includes data on a large number of products across four supermarket chains in 5 regions for 156 weeks.

Campina/Friesland

- Let's turn to the specification considered

$$w_{imt} = \sum_j \gamma_{ij} \ln p_{jmt} + \beta_j g_{mt} + trend1_t + trend2_t + c_q + c_m + \varepsilon_{imt}$$

- i is the brand subscript, m is the chain one and t is time (week)
- Where w and $\ln(p)$ are the expenditure shares and log of prices, g is the real segment specific expenditure.
- $trend1$ and $trend2$ are time trends before and after April 2007, c_q and c_m are the quarter and the panel (chain) fixed effect.
- The linear system of equations can be estimated by OLS. A more efficient method is the SUR (seemingly unrelated regressions) estimator, which applies FGLS to the system.

Campina/Friesland

- This amounts to taking into account the correlation between the error terms of the different equations.
- The SUR estimator uses the variance covariance matrix of the residuals from the OLS estimated equations as the weighting matrix in the FGLS estimation
- Another set of estimations is run using three stage least squares.
- Basically it applies the same logic of FGLS, although the residual used to construct the weighting matrix are those of a two stage least squares (IV estimation)
- Two panel estimators employed: FE estimators (where cm are dummies), or First-Difference estimator, where all variables are first differenced.